Received: *June 25, 2001*, Accepted: *September 10, 2001*
Revised: *September 10, 2001*

Is right-handed neutrino degeneracy compatible with the solar and atmospheric neutrino data?

Ricardo González Felipe and Filipe R. Joaquim

Centro de Física das Interacções Fundamentais, Departamento de Física
Instituto Superior Técnico, 1049-001 Lisboa, Portugal
E-mail: gonzalez@gtae3.ist.utl.pt, filipe@cfif3.ist.utl.pt

Abstract: In light of the recent solar and atmospheric neutrino data, we investigate the possibility of having an exactly degenerate spectrum for heavy right-handed Majorana neutrinos at the grand unification scale. The analysis is performed in the context of the minimal supersymmetric standard model with unbroken R parity and extended with three heavy Majorana neutrino fields in order to implement the seesaw mechanism. In the absence of a Dirac-type leptonic mixing, the only source of lepton flavour violation is the right-handed neutrino sector. Inspired by GUT-motivated relations among the quark, charged-lepton and Dirac neutrino Yukawa coupling matrices, and after the inclusion of the radiative effects, we determine the effective neutrino mass matrix at the electroweak scale. Using then the latest global analyses of the solar and atmospheric data at 99% C.L., we conclude that, within this framework, the only solar solutions compatible with the experimental data are the LOW and LMA solutions, being the latter the most favoured one. At 90% C.L., only the LMA solution is allowed.

Keywords: Solar and Atmospheric Neutrinos, Neutrino Physics.

JHEP09(2001)015

Contents

1. Introduction **1**

2. Theoretical framework **4**

3. The effects of radiative corrections **6**
- 3.1 Running $\mathbf{Y}_\nu$ from Λ_X to Λ_R 6
- 3.2 Running $\mathbf{M}_R$ from Λ_X to Λ_R 7
- 3.3 $\mathbf{M}_\nu^{\text{eff}}$ from high to low scales 8

4. Confronting the neutrino mass matrix with the experimental data **11**
- 4.1 Neutrino oscillation data 11
- 4.2 Neutrinoless double $\boldsymbol{\beta}$ decay 14

5. A simple analytical form for $\mathbf{M}_\nu^{\text{eff}}$ **16**

6. Conclusions **19**

A. Renormalization group equations **21**

JHEP09(2001)015

1. Introduction

The recent observation of solar and atmospheric neutrino deficits constitutes a solid evidence for physics beyond the Standard Model. The existence of such anomalies gives us a strong indication of neutrino oscillations among the three known families, which in turn imply that neutrinos have nonvanishing masses and nontrivial mixings. The latest atmospheric neutrino data from the Super-Kamiokande collaboration [1] and CHOOZ experiments [2] tends to favour the $\nu_\mu \to \nu_\tau$ oscillations with a large mixing angle $\theta_{\text{atm}} \simeq \pi/4$. On the other hand, the solar neutrino experiments [3]–[6] point towards the conversions $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$, and strongly favour the large mixing angle (LMA) MSW [7] solution over the small mixing angle (SMA), the LOW and the just-so vacuum oscillation (VO) solutions, as well as over the sterile neutrino hypothesis.

From a theoretical viewpoint, modelling neutrino masses and mixings represents a challenge. First, the presently most favoured solution, namely the LMA MSW solution, is usually the most difficult one to achieve in realistic models. Second, large mixing angles do not seem to fit naturally in our grand unification view. This is because in a very large class of string and grand unified theories (GUT's), the quark, charged lepton and Dirac neutrino mass matrices are typically related to each other and, therefore, one would expect small neutrino mixings in analogy to what is observed in the quark sector. The above reasoning assumes of course that the large neutrino mixing arises only from the Dirac neutrino mass matrix structure. Third, even if large neutrino mixings are consistently incorporated in the general picture of fermions, a natural question then arises, namely, why neutrino masses are so tiny compared to the rest of the fermions of the theory.

A simple and economical way to overcome some of the above difficulties is to invoke the so-called seesaw mechanism [8] which, in the presence of heavy right-handed Majorana neutrinos, ν_R, leads to an effective mass matrix for the light neutrinos. Rather than an assumption, the existence of the ν_R's is a natural requirement in the context of all GUT's with a group symmetry larger than SU(5). The seesaw mechanism by itself has, unfortunately, a very limited predictive power: the overall scale of the light neutrino masses is not uniquely determined, the mixing angles and the ratios of the effective neutrino masses are not fixed.

Grand unified theories such as SO(10) [9] are a suitable framework not only to analyze fermion masses but also to implement the seesaw mechanism. One of the attractive features of the SO(10) model is that its gauge group is left-right symmetric and, consequently, there exists a complete quark-lepton symmetry in the spectrum. In particular, the fact that all left-handed (right-handed) fermions of each family fit into the single irreducible spinor representation **16** ($\overline{\mathbf{16}}$) of SO(10) and that the right-handed neutrino ν_R is precisely contained in this representation is remarkable. As it happens in all gauge groups, after the spontaneous symmetry breaking, the fermions will acquire masses out of the Yukawa couplings and the vacuum expectation values (VEV) of the Higgs fields. Several constraints on fermion masses are usually implied in these models [9]. For instance, if there is only one **10** Higgs multiplet responsible for the masses, then we have the relation $\mathbf{Y}_U = \mathbf{Y}_D = \mathbf{Y}_\ell = \mathbf{Y}_\nu$ (the indices U, D, ℓ and ν stand for the up quarks, down quarks, charged leptons and Dirac neutrinos, respectively). Similarly, the existence of two **10** Higgs multiplets implies $\mathbf{Y}_D = \mathbf{Y}_\ell$ and $\mathbf{Y}_U = \mathbf{Y}_\nu$. On the other hand, if the fermion masses are generated by a VEV of the **126** of SO(10), then the SU(4) symmetry yields the relations $3\,\mathbf{Y}_D = -\mathbf{Y}_\ell$ and $3\,\mathbf{Y}_U = -\mathbf{Y}_\nu$. Of course, equalities such as the ones arising purely from the **10**-dimensional representation cannot be exact in realistic models, since they imply undesirable relations among the quark and charged lepton masses. Additional assumptions are therefore necessary in order to predict the correct fermion spectrum [10, 11].

At this point it is worth recalling that the Majorana mass matrix, $\mathbf{M}_R$, for the right-handed neutrinos is usually much less constrained in unified models than the Dirac mass matrix. This has led many authors to discuss possible textures for the heavy Majorana neutrino mass matrix within the seesaw framework [12]–[17]. A common feature to most of these analyses is the fact that the heavy Majorana spectrum turns out to be either hierarchical or partially degenerate. We may therefore ask ourselves whether a complete degenerate spectrum is allowed for $\mathbf{M}_R$ in this context. Similar studies have been previously performed in the case of degenerate left-handed Majorana neutrinos [18]–[20]. In particular, it has recently been argued [19] that if the Majorana masses of the light neutrinos are exactly degenerate at some high scale, then the one-loop radiative corrections are not sufficient to produce the required neutrino mass splittings and mixings at low scale.

Although a fully degenerate mass pattern for the light neutrinos could be natural, if the Majorana masses arise from nonrenormalizable operators, and is welcome by cosmology, e.g. if relic neutrinos constitute the hot dark matter of the Universe, at a fundamental level it seems more plausible to construct and motivate patterns for the Dirac and right-handed Majorana matrices rather than for the effective neutrino mass matrix. In fact, the effective neutrino mass matrix is only a secondary output of the seesaw mechanism and, consequently, it may not be directly related to any symmetry of the theory.

Of course, the observed hierarchy $\Delta m^2_{\rm atm} = |m_3^2 - m_2^2| \gg \Delta m^2_\odot = |m_2^2 - m_1^2|$ among the light neutrino masses m_i $(i = 1, 2, 3)$ implies that one of the following mass patterns should be realized: $|m_{1,2}| \ll |m_3|$ (hierarchical), $|m_1| \sim |m_2| \gg |m_3|$ (inversely hierarchical) or $|m_1| \sim |m_2| \sim |m_3|$ (almost degenerate). Thus, if one believes in the seesaw as the mechanism responsible for the smallness of neutrino masses, any realistic texture for the Dirac and Majorana matrices should lead to one of these patterns.

In this paper we address the question whether right-handed neutrino degeneracy is compatible or not with the present experimental solar and atmospheric neutrino data. We assume complete degeneracy of the heavy Majorana neutrinos ν_R at GUT scale and also that the right-handed neutrino sector provides all the source of lepton flavour violation in the theory. Neglecting the rotation between the charged lepton and the Dirac neutrino mass matrices is a reasonable approximation, if the Higgs field has its components along suitable representations of SO(10) such as the ones mentioned above. This scenario is quite appealing since one starts from a situation where there is no leptonic mixing (similar to what happens in the quark sector where the mixing is described by the CKM matrix and is therefore small) to generate large mixing through the seesaw mechanism.

When discussing neutrino masses and mixing angles, a crucial aspect is the running of the various couplings from the unification scale down to the electroweak scale. In the presence of nonvanishing neutrino couplings, the renormalization group equations (RGE) are modified. From the unification scale Λ_X to the ν_R decoupling scale Λ_R, the radiative corrections from the Dirac and right-handed Majorana neutrinos must be included. Below Λ_R, when the seesaw mechanism becomes operative and

JHEP09(2001)015

the ν_R's decouple from the theory, the running of the effective neutrino Yukawa couplings then becomes relevant. Radiative corrections could indeed provide potential contributions to the effective neutrino mass matrix or be even disastrous. A typical example is given by the RGE effects due to the charged-lepton Yukawa couplings [21].

The outline of the paper is as follows. In section 2 we briefly present the general framework used in our analysis, with some attention given to the parametrization of the degenerate right-handed Majorana neutrino mass matrix. Section 3 is devoted to the study of the RGE effects from the GUT scale down to the electroweak scale. In order to account for the right-handed decoupling threshold at Λ_R, the RGE analysis consists of two steps: first, the running of all the quark, charged-lepton as well as the Dirac and right-handed neutrino Yukawa couplings from Λ_X to Λ_R and, second, the evolution of all the quark and charged-lepton couplings together with the effective neutrino mass matrix from Λ_R to the electroweak scale M_Z. Such an analysis is required in order to verify whether the imposed patterns for the Yukawa coupling matrices at high scales are compatible or not with the low-energy data. In section 4, the effective neutrino mass matrix, obtained at the electroweak scale, is confronted with the experimental data to determine which of the currently allowed solar neutrino solutions can indeed be realized in the present framework. Some numerical examples for the most favoured solution, namely the LMA MSW solution, and for different hierarchical patterns of the Dirac neutrino Yukawa couplings, are given in this section. Few comments on neutrinoless double β decay are also presented. These results allow us to motivate a simple analytical structure for the effective neutrino mass matrix in section 5. Finally, we present our concluding remarks in section 6.

2. Theoretical framework

We shall work in the context of the minimal supersymmetric standard model (MSSM) with unbroken R parity and extended with 3 heavy right-handed neutrinos. Supersymmetry (SUSY) is a natural choice to implement the seesaw mechanism. For instance, the hierarchy problem, usually present in a nonsupersymmetric seesaw, is automatically cured in the supersymmetric case. Indeed, the existence of very heavy right-handed neutrinos coupled to the Higgs field implies logarithmically-divergent radiative corrections to the Higgs mass $\sim \Lambda_R^2 \mathbf{Y}_\nu^2 H^2 \ln(\Lambda_R^2/\mu^2)$. While such corrections persist in nonsupersymmetric theories, they are naturally cancelled in SUSY by similar contributions coming from the right-handed sneutrinos.

The relevant Yukawa mass terms are described by the lagrangian (generation indices are suppressed)

$$\begin{aligned}\mathcal{L}_Y =& \bar{Q}\,\mathbf{Y}_U\,U^c\,H_2 + \bar{Q}\,\mathbf{Y}_D\,D^c\,H_1 + \bar{L}\,\mathbf{Y}_\ell\,E^c\,H_1 + \\ &+ \bar{L}\,\mathbf{Y}_\nu\,\nu_R^c\,H_2 + \frac{1}{2}\,\nu_R^{c\,T}\,\mathbf{M}_R\,\nu_R^c + \text{h.c.}\,, \end{aligned} \tag{2.1}$$

where $\mathbf{Y}_U$, $\mathbf{Y}_D$, $\mathbf{Y}_\ell$ and $\mathbf{Y}_\nu$ are the up-quark, down-quark, charged-lepton and Dirac neutrino Yukawa coupling matrices,[1] respectively; $\mathbf{M}_R$ is a 3×3 symmetric Majorana mass matrix which preserves the SM gauge symmetry; Q and L are the quark and charged-lepton left-handed doublets; U^c, D^c and E^c are the up-quark, down-quark and charged-lepton right-handed singlets. The additional neutrino chiral fields ν_{Ri} $(i = e, \mu, \tau)$ are right-handed Majorana fields responsible for the seesaw mechanism. Finally, H_1 and H_2 are the hypercharge $-1/2$ and $+1/2$ MSSM Higgs doublets, whose neutral components acquire VEVs after the spontaneous symmetry breaking: $\langle H_1^0 \rangle = v\cos\beta$, $\langle H_2^0 \rangle = v\sin\beta$, with $v \simeq 174\,\mathrm{GeV}$.

In what follows we shall assume that $\mathbf{Y}_\nu \propto \mathbf{Y}_U$ and also that there is a left-handed alignment between $\mathbf{Y}_\ell$ and $\mathbf{Y}_\nu$ at the unification scale Λ_X, which can be equal or higher than the decoupling scale Λ_R , i.e. the scale at which the seesaw mechanism becomes operative and the ν_R's decouple from the light fields. As mentioned in the Introduction, these are reasonable assumptions in the context of grand unified theories. This means, in particular, that the only source of non-trivial leptonic mixing is the right-handed neutrino sector. In this scenario large neutrino mixings will be generated through the seesaw mechanism.

Next we assume that the right-handed Majorana neutrinos are degenerate at Λ_X. In this case a simple and general parametrization [22] can be given for the right-handed Majorana neutrino mass matrix $\mathbf{M}_R$. Indeed, since $\mathbf{M}_R$ is symmetric we can always write

$$W^T\,\mathbf{M}_R\,W = \Lambda_R \mathbb{1}\,, \qquad \mathbf{Y}_R \equiv \Lambda_R^{-1}\mathbf{M}_R = W^* W^\dagger\,, \tag{2.2}$$

where W is a unitary matrix and $\mathbf{Y}_R$ is a unitary symmetric matrix. It is then straightforward to show that the matrix $\mathbf{Y}_R$ can be parametrized as

$$\mathbf{Y}_R = \mathbf{R}_\theta\,\mathbf{R}_\phi\,\mathbf{R}_\theta{}^T\,, \tag{2.3}$$

where the rotation matrices $\mathbf{R}_\theta$ and $\mathbf{R}_\phi$ are given by

$$\mathbf{R}_\theta = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{R}_\phi = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & c_{2\phi} & -s_{2\phi} \\ 0 & -s_{2\phi} & -c_{2\phi} \end{pmatrix}. \tag{2.4}$$

Here we have introduced, for simplicity, the notations $c_\varphi \equiv \cos\varphi$, $s_\varphi \equiv \sin\varphi$.

Let us note that the parametrization (2.3) does not include the trivial case when CP is conserved and all the right-handed neutrinos have the same CP parity [22]. The CP-violating phase α turns out to be irrelevant in our further analysis and thus we shall set $\alpha = 0$. In the latter case the complete form of $\mathbf{Y}_R$ is given by

$$\mathbf{Y}_R = \begin{pmatrix} c_\theta^2 + c_{2\phi}s_\theta^2 & -s_{2\theta}s_\phi^2 & -s_\theta s_{2\phi} \\ -s_{2\theta}s_\phi^2 & s_\theta^2 + c_{2\phi}c_\theta^2 & -c_\theta s_{2\phi} \\ -s_\theta s_{2\phi} & -c_\theta s_{2\phi} & -c_{2\phi} \end{pmatrix}. \tag{2.5}$$

[1]We assume all the Yukawa coupling matrices real.

Moreover,

$$\mathbf{M}_R{}^{-1} = \Lambda_R^{-1}\,[\,\mathbf{R}_\theta\,\mathbf{R}_\phi\,\mathbf{R}_\theta{}^T\,]^{-1} = \Lambda_R^{-1}\,\mathbf{R}_\theta\,\mathbf{R}_\phi\,\mathbf{R}_\theta{}^T = \Lambda_R^{-1}\,\mathbf{Y}_R\,. \tag{2.6}$$

Notice that the parametrization (2.5) highly constrains the form of $\mathbf{M}_R$, which only depends on two angles θ, ϕ and the decoupling scale Λ_R .

3. The effects of radiative corrections

In this section we shall consider the evolution of the couplings from the unification scale Λ_X to the electroweak scale M_Z. The relevant one-loop RGE, which include neutrino threshold effects, are summarized in the appendix. Let us note that it is customary to take the SUSY breaking scale as $\Lambda_{\rm SUSY} \simeq 1\,\mathrm{TeV}$ or $\Lambda_{\rm SUSY} \simeq m_t$. Since in general the renormalization group effects that could possibly arise between $\Lambda_{\rm SUSY}$ and M_Z are negligible, we will not account for such effects in our analysis. Thus we set $\Lambda_{\rm SUSY} = M_Z$.

At the unification scale Λ_X the Yukawa matrices $\mathbf{Y}_\ell$ and $\mathbf{Y}_\nu$ can be diagonalized by real and orthogonal matrices $U_{L,R}$ and $V_{L,R}$ such that

$$\begin{aligned} U_L{}^\dagger\,\mathbf{Y}_\ell\,U_R &= \mathrm{diag}\,(y_e(\Lambda_X)\,,\,y_\mu(\Lambda_X)\,,\,y_\tau(\Lambda_X))\,,\\ V_L{}^\dagger\,\mathbf{Y}_\nu\,V_R &= \mathrm{diag}\,(y_1(\Lambda_X)\,,\,y_2(\Lambda_X)\,,\,y_3(\Lambda_X))\,, \end{aligned} \tag{3.1}$$

where $y_i(\Lambda_X)$ denotes the respective Yukawa couplings at Λ_X. The hypothesis of left-handed alignment between $\mathbf{Y}_\nu$ and $\mathbf{Y}_\ell$ implies $U_L = V_L$. In this case, $\mathbf{Y}_\nu$ and $\mathbf{Y}_\ell$ can be simultaneously diagonalized, leaving the charged-current lagrangian invariant. This means that there is no "Dirac-type" leptonic mixing at Λ_X. Obviously, this diagonalization process does not affect the spectrum of the right-handed Majorana matrix and, therefore, the resulting matrix $\mathbf{M}'_R = V_L^T\,\mathbf{M}_R\,V_L$ can also be parametrized in the form of eq. (2.5) with a different set of the parameters θ and ϕ.

3.1 Running $\mathbf{Y}_\nu$ from Λ_X to Λ_R

Our first step in discussing the effects of radiative corrections on neutrino masses and mixing angles will be the study of the evolution of the Dirac Yukawa coupling matrix $\mathbf{Y}_\nu$ from GUT scale to the right-handed neutrino decoupling scale. After performing the diagonalization given in eq. (3.1) we have

$$\mathbf{Y}_\nu\,(\Lambda_X) = y_3(\Lambda_X)\begin{pmatrix}\epsilon_1 & 0 & 0\\ 0 & \epsilon_2 & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{3.2}$$

where the parameters

$$\epsilon_1 = \frac{y_1(\Lambda_X)}{y_3(\Lambda_X)}\,, \qquad \epsilon_2 = \frac{y_2(\Lambda_X)}{y_3(\Lambda_X)}\,, \tag{3.3}$$

have been introduced.

The running of $\mathbf{Y}_\nu$ from Λ_X to Λ_R is governed by eq. (A.4). From this equation and neglecting all but the third-generation Yukawa couplings[2] we get

$$\mathbf{Y}_\nu(\Lambda_R) \simeq I_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_\nu \end{pmatrix} \mathbf{Y}_\nu(\Lambda_X)\,, \tag{3.4}$$

where

$$I_1 = \exp\left[\frac{1}{16\,\pi^2}\int_{t_X}^{t_R} (\,T_\nu - G_\nu)\,dt\right] \simeq \exp\left[\frac{1}{16\,\pi^2}\int_{t_X}^{t_R} \left(3y_t^2 + y_3^2 - \frac{3}{5}g_1^2 - 3g_2^2\right)\,dt\right], \tag{3.5}$$

$$I_\nu \simeq \exp\left[\frac{1}{16\,\pi^2}\int_{t_X}^{t_R} (\,3y_3^2 + y_\tau^2\,)\,dt\right], \tag{3.6}$$

with $t_X \equiv \ln(\Lambda_X/M_Z)$ and $t_R \equiv \ln(\Lambda_R/M_Z)$.

From eqs. (3.2) and (3.4) we find

$$\mathbf{Y}_\nu(\Lambda_R) = K_D \begin{pmatrix} \epsilon_1' & 0 & 0 \\ 0 & \epsilon_2' & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad K_D = y_3(\Lambda_X)\,I_1\,I_\nu\,, \qquad \epsilon_{1,2}' = I_\nu^{-1}\,\epsilon_{1,2}\,. \tag{3.7}$$

3.2 Running $\mathbf{M}_R$ from Λ_X to Λ_R

The running of the right-handed Majorana mass matrix $\mathbf{M}_R$ from Λ_X to Λ_R is described by eq. (A.10) given in the appendix. This equation allows us to relate the right-handed neutrino mass matrices $\mathbf{M}_R(\Lambda_R)$ and $\mathbf{M}_R(\Lambda_X)$:

$$[\mathbf{M}_R(\Lambda_R)]_{ij} = [\mathbf{M}_R(\Lambda_X)]_{ij}\,\exp\left[\frac{1}{8\,\pi^2}\int_{t_X}^{t_R}\left(y_i^2 + y_j^2\right)\,dt\right]. \tag{3.8}$$

In the limit where only the third Dirac neutrino Yukawa coupling y_3 is kept, we find

$$\mathbf{M}_R(\Lambda_R) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_R \end{pmatrix} \mathbf{M}_R(\Lambda_X) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_R \end{pmatrix}, \tag{3.9}$$

where

$$I_R \simeq \exp\left[\frac{1}{8\,\pi^2}\int_{t_X}^{t_R} y_3^2\,dt\right]. \tag{3.10}$$

The inverse of M_R at Λ_R is then given by

$$\mathbf{M}_R^{-1}(\Lambda_R) = \Lambda_R^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_R^{-1} \end{pmatrix} \mathbf{Y}_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_R^{-1} \end{pmatrix}, \tag{3.11}$$

where $\mathbf{Y}_R$ has the form given in eq. (2.5).

[2]This is a reasonable assumption considering that we will focus on cases where $y_1 \ll y_2 \ll y_3$.

3.3 $\mathbf{M}_\nu^{\rm eff}$ from high to low scales

In order to study the low-energy consequences of our assumptions, imposed at high scales, we must find the effective neutrino mass matrix $\mathbf{M}_\nu^{\rm eff}$ at the typical neutrino experiment scale, i.e. the electroweak scale ($\simeq M_Z$). If R parity is an exact symmetry in the MSSM, the lowest (5-dimensional) operator in the superpotential which violates lepton number and generates a Majorana mass for the left-handed neutrinos, ν, is given by [23, 24]

$$W_{\nu\nu} = \frac{1}{4}\boldsymbol{\kappa}\,\nu^T\,\nu\,H_2^0\,H_2^0 + \text{h.c.}\,, \tag{3.12}$$

where $\boldsymbol{\kappa}$ is a 3×3 symmetric matrix. The effective neutrino mass matrix is obviously given by

$$\mathbf{M}_\nu^{\rm eff} = \boldsymbol{\kappa}\langle H_2^0\rangle^2 = \boldsymbol{\kappa}\,v^2\sin^2\beta\,. \tag{3.13}$$

The supersymmetric seesaw mechanism is realized by adding to the superpotential the following terms:

$$W = W_{\rm MSSM} - \frac{1}{2}\nu_R^{c\,T}\,\mathbf{M}_R\,\nu_R^c + \nu_R^{c\,T}\mathbf{Y}_\nu\,L\,H_2\,. \tag{3.14}$$

Below Λ_R one can consider the effective superpotential given by

$$W_{\rm eff} = W_{\rm MSSM} + \frac{1}{2}(\mathbf{Y}_\nu\,L\,H_2)^T\,\mathbf{M}_R^{-1}\,(\mathbf{Y}_\nu\,L\,H_2)\,, \tag{3.15}$$

which is obtained by integrating out the heavy right-handed neutrino fields ν_R in eq. (3.14). The second term in $W_{\rm eff}$ will originate in the lagrangian a Majorana mass term of the type

$$\mathcal{L}_{\nu\nu} = -\frac{1}{2}\,\nu^T\boldsymbol{\kappa}\,\nu\,H_2^0\,H_2^0 + \text{h.c.}\,, \tag{3.16}$$

where

$$\boldsymbol{\kappa} = \mathbf{Y}_\nu^T\,\mathbf{M}_R^{-1}\,\mathbf{Y}_\nu\,. \tag{3.17}$$

Let us note that at this stage the matrices $\boldsymbol{\kappa}$ and $\mathbf{M}_\nu^{\rm eff}$ are defined at the decoupling scale Λ_R. To compare them with the experimental ones, obtained at the electroweak scale, we must run these matrices down to M_Z through the corresponding RGE. Taking into account eqs. (3.7) and (3.11) we have

$$\boldsymbol{\kappa}(\Lambda_R) = \Lambda_R^{-1}\,\mathbf{Y}_\nu^T(\Lambda_R)\,\mathbf{Y}_R\,\mathbf{Y}_\nu(\Lambda_R) = \frac{K_D^2}{\Lambda_R}\begin{pmatrix}\epsilon_1' & 0 & 0\\ 0 & \epsilon_2' & 0\\ 0 & 0 & I_R^{-1}\end{pmatrix}\mathbf{Y}_R\begin{pmatrix}\epsilon_1' & 0 & 0\\ 0 & \epsilon_2' & 0\\ 0 & 0 & I_R^{-1}\end{pmatrix}. \tag{3.18}$$

JHEP09(2001)015

The running of the matrix $\boldsymbol{\kappa}$ from Λ_R to M_Z is described by the RGE given in eq. (A.14). From the latter equation, and neglecting the charged-lepton Yukawa couplings y_μ and y_e, we get the following relation between $\boldsymbol{\kappa}(M_Z)$ and $\boldsymbol{\kappa}(\Lambda_R)$:

$$\boldsymbol{\kappa}(M_Z) = I_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_\tau \end{pmatrix} \boldsymbol{\kappa}(\Lambda_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_\tau \end{pmatrix}, \tag{3.19}$$

where

$$I_2 = \exp\left[\frac{1}{16\,\pi^2}\int_{t_R}^{t_Z} (\,T_{\boldsymbol{\kappa}} - G_{\boldsymbol{\kappa}})\,dt\right] \simeq \exp\left[\frac{1}{16\,\pi^2}\int_{t_R}^{t_Z}\left(6y_t^2 - \frac{6}{5}g_1^2 - 6g_2^2\right)\,dt\right], \tag{3.20}$$

$$I_\tau = \exp\left[\frac{1}{16\,\pi^2}\int_{t_R}^{t_Z} y_\tau^2\,dt\right]. \tag{3.21}$$

Using eq. (3.18) we finally obtain

$$\boldsymbol{\kappa}(M_Z) = \frac{K_{\rm eff}}{\Lambda_R}\begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{Y}_R(\Lambda_X) \begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.22}$$

with

$$\delta_{1,2} = I_{\rm eff}\, I_\tau^{-1}\,\epsilon_{1,2}\ , \quad I_{\rm eff} = I_R\, I_\nu^{-1}\ , \tag{3.23}$$
$$K_{\rm eff} = y_3^2(\Lambda_X)\, I_1^2\, I_2\, I_\tau^2\, I_{\rm eff}^{-2}\ , \tag{3.24}$$

and I_1, I_2 given in eqs. (3.5) and (3.20), respectively.

From eqs. (2.5) and (3.22) we obtain our final expression for the neutrino effective operator at the electroweak scale:

$$\boldsymbol{\kappa}(M_Z) = \frac{K_{\rm eff}}{\Lambda_R}\begin{pmatrix} \delta_1^2\,(c_\theta^2 + c_{2\phi}s_\theta^2) & -\delta_1\,\delta_2\,s_{2\theta}s_\phi^2 & -\delta_1\,s_\theta s_{2\phi} \\ -\delta_1\,\delta_2\,s_{2\theta}s_\phi^2 & \delta_2^2\,(s_\theta^2 + c_{2\phi}c_\theta^2) & -\delta_2\,c_\theta s_{2\phi} \\ -\delta_1\,s_\theta s_{2\phi} & -\delta_2\,c_\theta s_{2\phi} & -c_{2\phi} \end{pmatrix}. \tag{3.25}$$

Let us remark that the mass spectrum and mixings arising from this matrix are invariant under the transformations $(\theta \to \theta,\ \phi \to \pi - \phi)$ and $(\theta \to \pi - \theta,\ \phi \to \phi)$, and, obviously, under any transformation resulting from their successive application. This simply corresponds to a phase redefinition of the left-handed neutrino fields. Thus, from now on we restrict ourselves to the intervals $0 \leq \theta, \phi \leq \pi/2$.

In the limit where the evolution of the third Dirac neutrino Yukawa coupling y_3 and the τ Yukawa coupling y_τ can be neglected, we find the approximate expressions:

$$I_\nu \simeq \left(\frac{\Lambda_R}{\Lambda_X}\right)^{\frac{3y_3^2+y_\tau^2}{16\pi^2}}, \qquad I_R \simeq \left(\frac{\Lambda_R}{\Lambda_X}\right)^{\frac{y_3^2}{8\pi^2}}, \qquad I_\tau \simeq \left(\frac{M_Z}{\Lambda_R}\right)^{\frac{y_\tau^2}{16\pi^2}}, \tag{3.26}$$

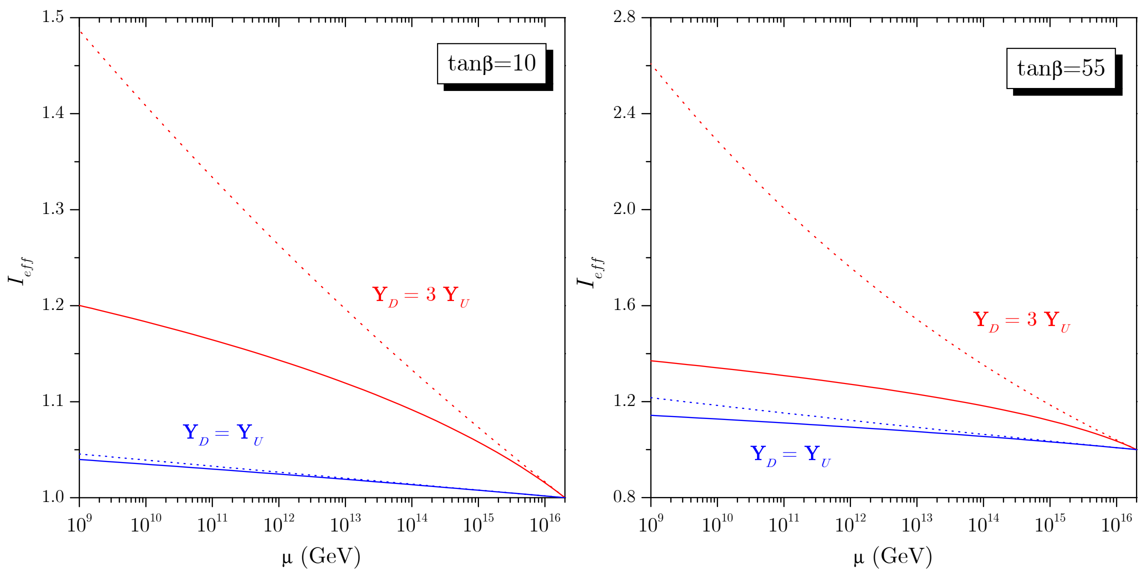

Figure 1: The evolution of the RGE coefficient $I_{\rm eff} = I_R I_\nu^{-1}$ from Λ_X to Λ_R, assuming two different Dirac neutrino textures, $\mathbf{Y}_D = \mathbf{Y}_U$ and $\mathbf{Y}_D = 3\mathbf{Y}_U$, and for the values of $\tan\beta = 10$ and $\tan\beta = 55$. The solid curves correspond to the exact expressions as defined by eqs. (3.6) and (3.10). The dashed lines correspond to the approximate curves as given in eqs. (3.26).

and, consequently,

$$\delta_{1,2} \simeq \left(\frac{\Lambda_X}{\Lambda_R}\right)^{\frac{y_3^2+y_\tau^2}{16\pi^2}} \left(\frac{\Lambda_R}{M_Z}\right)^{\frac{y_\tau^2}{16\pi^2}} \epsilon_{1,2}\,, \tag{3.27}$$

$$K_{\rm eff} \simeq y_3^2(\Lambda_X)\, I_1^2\, I_2 \left(\frac{\Lambda_R}{\Lambda_X}\right)^{\frac{y_3^2+y_\tau^2}{8\pi^2}} \left(\frac{M_Z}{\Lambda_R}\right)^{\frac{y_\tau^2}{8\pi^2}}\,. \tag{3.28}$$

At this point few remarks are in order. First, as it has already been noticed by several authors [21], the parameter $I_\tau \simeq 1$. This can be easily checked by using either the exact expression (3.21) or the approximate expression given in eq. (3.26). Moreover, the parameter $I_{\rm eff}$, which controls the RGE effects of the Dirac neutrino Yukawa coupling ratios (cf. eq. (3.23)), is also close to the unity. This can be seen from figure 1 where we present the dependence of $I_{\rm eff}$ on the value of the scale μ for the two initial conditions $\mathbf{Y}_\nu = \mathbf{Y}_U$ and $\mathbf{Y}_\nu = 3\,\mathbf{Y}_U$. We note that $I_{\rm eff} \simeq 1$ for a wide range of $\tan\beta$. Notice also that for the case $\mathbf{Y}_\nu = 3\,\mathbf{Y}_U$, the use of the approximate expression for $I_{\rm eff}$, obtained by neglecting the running of the Yukawa couplings, is not well justified. This is because in the latter case $y_3(\Lambda_X) = 3y_t(\Lambda_X)$ and the evolution of the couplings becomes more relevant.

4. Confronting the neutrino mass matrix with the experimental data

The analysis of the neutrino oscillation data indicates that, contrary to what happens in the quark sector, where the mixing is always small, the mixing angle is large in the 2-3 neutrino sector. The latest atmospheric neutrino data from the Super-Kamiokande collaboration [1] and CHOOZ experiments [2] implies a large mixing angle $\theta_{\rm atm} \simeq \pi/4$. In what concerns the 1-2 neutrino sector, there are four different scenarios depending on the solar neutrino problem solution.

Since the present experimental data constrains the neutrino mass squared differences and the mixing angles, it is necessary to diagonalize the effective neutrino mass matrix $\mathbf{M}_\nu^{\rm eff}$ at the electroweak scale. The neutrino mixing matrix $\mathbf{U}$, which relates the flavour and mass eigenstates,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \mathbf{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{4.1}$$

can be parametrized in the standard form [25]

$$\mathbf{U} = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} \end{pmatrix}, \tag{4.2}$$

such that $\mathbf{U}^T\,\mathbf{M}_\nu^{\rm eff}(M_Z)\,\mathbf{U} = {\rm diag}(m_1, m_2, m_3)$, where m_i are the light neutrino masses; $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$.

We also remark that the lepton mixing matrix can have, besides a Dirac-type phase (analogous to that of the quark sector), two other physical phases associated with the Majorana character of neutrinos. If CP is conserved these phases are either 0 or π. In the analysis that follows we shall assume all these phases vanishing. However, as we shall see in section 4.2, such phases play a significant role in processes such as the neutrinoless double β decay [26].

4.1 Neutrino oscillation data

The relevant parameter set for the study of solar and atmospheric data is usually identified with the quantities

$$\begin{aligned} &\Delta m^2_\odot \equiv \Delta m^2_{12} = m_2^2 - m_1^2\,, \qquad \Delta m^2_{\rm atm} \equiv \Delta m^2_{23} = m_3^2 - m_2^2\,, \\ &\tan^2\theta_\odot \equiv \tan^2\theta_{12}\,, \qquad \tan^2\theta_{\rm atm} \equiv \tan^2\theta_{23}\,, \qquad U_{e3} \equiv \sin\theta_{13}\,, \end{aligned} \tag{4.3}$$

where all mixing angles are assumed to lie in the range $[0, \pi/2]$. In table 1 we summarize the present constraints on neutrino mass squared differences and on the mixing

	Atmospheric and reactor neutrinos		
	$\Delta m^2_{\rm atm}$ (eV^2)	$\tan^2\theta_{\rm atm}$	$\lvert U_{e3}\rvert$
	$(1.1 - 7.3)\times 10^{-3}$	0.3 - 3.8	< 0.27
	$[1.4 - 6.1]\times 10^{-3}$	[0.4 - 3.1]	$[< 0.23]$
	$\mathbf{3.1 \times 10^{-3}}$	**1.4**	**0.07**
	Solar neutrinos		
	$\Delta m^2_{\odot}$ (eV^2)	$\tan^2\theta_{\odot}$	$r_{\odot} = \Delta m^2_{\odot}/\Delta m^2_{\rm atm}$
LMA	$(2.0 - 20)\times 10^{-5}$	0.1 - 0.7	$(0.03 - 1.8)\times 10^{-1}$
	$[2.3 - 10]\times 10^{-5}$	[0.2 - 0.5]	$[0.04 - 0.7]\times 10^{-1}$
	$\mathbf{4.2 \times 10^{-5}}$	**0.26**	**0.014**
SMA	$(0.4 - 1)\times 10^{-5}$	$(0.2 - 2)\times 10^{-3}$	$(0.55 - 9.1)\times 10^{-3}$
LOW	$(0.5 - 2)\times 10^{-7}$	$0.5 - 0.9$	$(0.07 - 1.8)\times 10^{-4}$
Just-So^2	$(5 - 8)\times 10^{-12}$	$0.5 - 2$	$(0.68 - 7.3)\times 10^{-9}$

Table 1: Constraints on neutrino masses and mixing angles at 99% C.L. [90% C.L.] coming from global analyses of the solar, atmospheric and reactor neutrino data [27, 28]. The numbers in bold correspond to the best-fit values.

angles obtained from the recent global analyses [27, 28] of the solar, atmospheric and reactor neutrino data at 99% C.L. [90% C.L.] .

Apart from the overall factor $K_{\rm eff}/\Lambda_R$, the effective neutrino operator $\boldsymbol{\kappa}(M_Z)$ defined in eq.(3.25) depends on the angles θ and ϕ, which parametrize the heavy right-handed neutrinos, and the parameters $\delta_{1,2}$. For the structure of $\boldsymbol{\kappa}$ to be compatible with the low-energy data, we should take into account not only all the constraints on $r_{\odot} = \Delta m^2_{\odot}/\Delta m^2_{\rm atm}$ and the mixing angles, but also the experimentally allowed region for the up-quark mass ratios, since within our assumptions, these ratios fix the parameters δ_1 and δ_2 at Λ_X:

$$\delta_1 \simeq \epsilon_1 = \frac{y_u(\Lambda_X)}{y_t(\Lambda_X)}\,, \qquad \delta_2 \simeq \epsilon_2 = \frac{y_c(\Lambda_X)}{y_t(\Lambda_X)}\,. \tag{4.4}$$

The above ratios can be determined by running the RGE from the electroweak scale M_Z to the unification scale Λ_X, which we assume to be $\Lambda_X \simeq 2\times 10^{16}$ GeV. Since these ratios are quite insensitive to the value of $\tan\beta$ in a wide region, we shall take $\tan\beta = 10$ as the input value in our analysis. Using the experimentally allowed values for the quark masses at the electroweak scale [29, 25], the following ranges are

JHEP09(2001)015

then obtained at Λ_X:

$$
\begin{aligned}
2.8\times10^{-6} \lesssim \delta_1 \lesssim 1.5\times10^{-5}\,, \\
9\times10^{-4} \lesssim \delta_2 \lesssim 4\times10^{-3}\,.
\end{aligned}
\tag{4.5}
$$

In our numerical study we randomly varied all the parameters in the ranges: $0 \leq \delta_{1,2} \leq 1$, $0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq \pi/2$. In figure 2 we plot the allowed region in the plane (δ_1, δ_2) for the three different MSW solar solutions (LMA, SMA, LOW) at 99% C.L. We assume that the right-handed Majorana neutrinos are exactly degenerate at the unification scale Λ_X. The dashed area delimited by the dotted lines corresponds to the allowed region for the Dirac neutrino coupling ratios, as defined by eqs. (4.5) and assuming $\mathbf{Y}_\nu \propto \mathbf{Y}_U$ at Λ_X. The intersection of the filled areas with the dashed area provides us with the points (δ_1, δ_2) compatible with the low-energy data and the initial conditions imposed at Λ_X. From the plots it is clear that the SMA solution is excluded. The LOW solution is barely compatible with the data at 99% C.L., and certainly excluded at 90% C.L. Finally, the LMA solution is the most favoured one, and at 90% C.L. it is the only one that survives.

The results in the parameter space (ϕ, θ) are shown in figure 2d at 99% C.L.. We notice that the allowed region for LMA is more extensive than the SMA and LOW regions, and the SMA region is in turn wider than the LOW one. This is a direct consequence of the hierarchy ${r_\odot}^{\rm LMA} > {r_\odot}^{\rm SMA} > {r_\odot}^{\rm LOW}$. For the VO (Just-So^2) solar solutions an extreme fine-tuning is required between θ and ϕ in order to get a solution compatible with the solar and atmospheric neutrino data. The latter solutions are therefore not presented in our plots.

Since the LMA solution is the most favoured in the present framework, the rest of our discussion will be devoted to it. In figures 3 we show the allowed region at 99% C.L. for the LMA MSW solution, in different planes of the input parameters δ_1, δ_2, ϕ and θ. A remarkable feature is the fact that for small values of δ_1 and δ_2, which are required in our framework, one has $\phi \simeq \pi/4$ and $\theta \simeq \pi/2$. We remark that the same conclusion for the value of ϕ can be drawn by looking at the structure of the effective neutrino mass matrix (3.25), since for small values of $\delta_{1,2}$, the relation $\cos 2\phi \ll 1$ is required to avoid a quasi-diagonal pattern in the mass matrix.

Let us conclude this section by giving few numerical examples (see table 2). We consider two different GUT-motivated relations for the Dirac neutrino Yukawa coupling matrix: $\mathbf{Y}_\nu = \mathbf{Y}_U$ and $\mathbf{Y}_\nu = 3\mathbf{Y}_U$. In both cases, consistency with the present LMA solar, atmospheric and reactor neutrino data was achieved. It was not possible however to obtain the best-fit values, simultaneously for all the parameters. By slightly modifying the initial assumption $y_1/y_3 = y_u/y_t$, we were then able to reproduce the best-fit values for all the solar and atmospheric neutrino data, as shown in the last column of the table. Notice also that the obtained value $|U_{e3}| \simeq 0.05$ is close to the best-fit value $|U_{e3}| = 0.07$ given in table 1.

As suggested by our full numerical analysis (see figures 3), the angles θ and ϕ were chosen to be close to $\pi/2$ and $\pi/4$, respectively. The degeneracy imposed at Λ_X for the right-handed Majorana neutrinos, $|M_1| = |M_2| = |M_3|$, is lifted in all cases by radiative corrections. At the decoupling scale Λ_R we have $|M_1| \simeq |M_2| \simeq I_R \Lambda_R < |M_3| \simeq \Lambda_R$. For the effective neutrinos with masses m_i, we obtain at the electroweak scale a hierarchical spectrum with

$$|m_1| < |m_2| \simeq \sqrt{\Delta m_{12}^2} < |m_3| \simeq \sqrt{\Delta m_{23}^2} \, . \tag{4.6}$$

Finally, we also notice the stability of the atmospheric and solar neutrino parameters, $r_\nu = \Delta m_{12}^2 / \Delta m_{23}^2$, $\tan^2 \theta_{12}$, $\tan^2 \theta_{23}$ and U_{e3} with the energy scale. In fact, from Λ_R to M_Z no significant running of these parameters is observed. This is related with the fact that $I_\tau \simeq 1$.

4.2 Neutrinoless double β decay

The Majorana nature of massive neutrinos can be traced by looking for processes where the total lepton charge L changes by two units, i.e. $\Delta L = 2$. The information coming from these type of events can be crucial not only to establish the nature of neutrinos but also to reveal some aspects about CP violation in the leptonic sector, to which neutrino oscillations are insensitive. In particular, if neutrinos have a Dirac signature, then the two extra Majorana phases which appear in the neutrino mixing matrix will be absent. Moreover, even if CP invariance holds, some information about the relative CP parities and neutrino mass spectrum can be extracted. One example of such processes is the neutrinoless double β decay ($\beta\beta_{0\nu}$ - decay) of the type

$$(A, Z) \rightarrow (A, Z + 2) + 2\, e^- \, , \tag{4.7}$$

which can occur through the exchange of massive Majorana neutrinos coupled to electrons. The decay rate is proportional to the so-called "effective Majorana mass parameter"

$$|\langle m \rangle| \; = \left| m_1 \, U_{e1}^2 + m_2 \, U_{e2}^2 + m_3 \, U_{e3}^2 \right| \, . \tag{4.8}$$

Although no experimental evidence has been found so far for this kind of processes, an upper bound has been obtained by the ^{76}Ge Heidelberg-Moscow experiment [30]:

$$|\langle m \rangle| \; < 0.35 \, \mathrm{eV} \, [\, 90\% \, \mathrm{C.L.} \,] \, . \tag{4.9}$$

If CP is conserved, then eq. (4.8) can be written as

$$|\langle m \rangle| \; = \left| \eta_1^{CP} m_1 \, U_{e1}^2 + \eta_2^{CP} \, m_2 \, U_{e2}^2 + \eta_3^{CP} \, m_3 \, U_{e3}^2 \right| \; , \quad m_i > 0 \; , \tag{4.10}$$

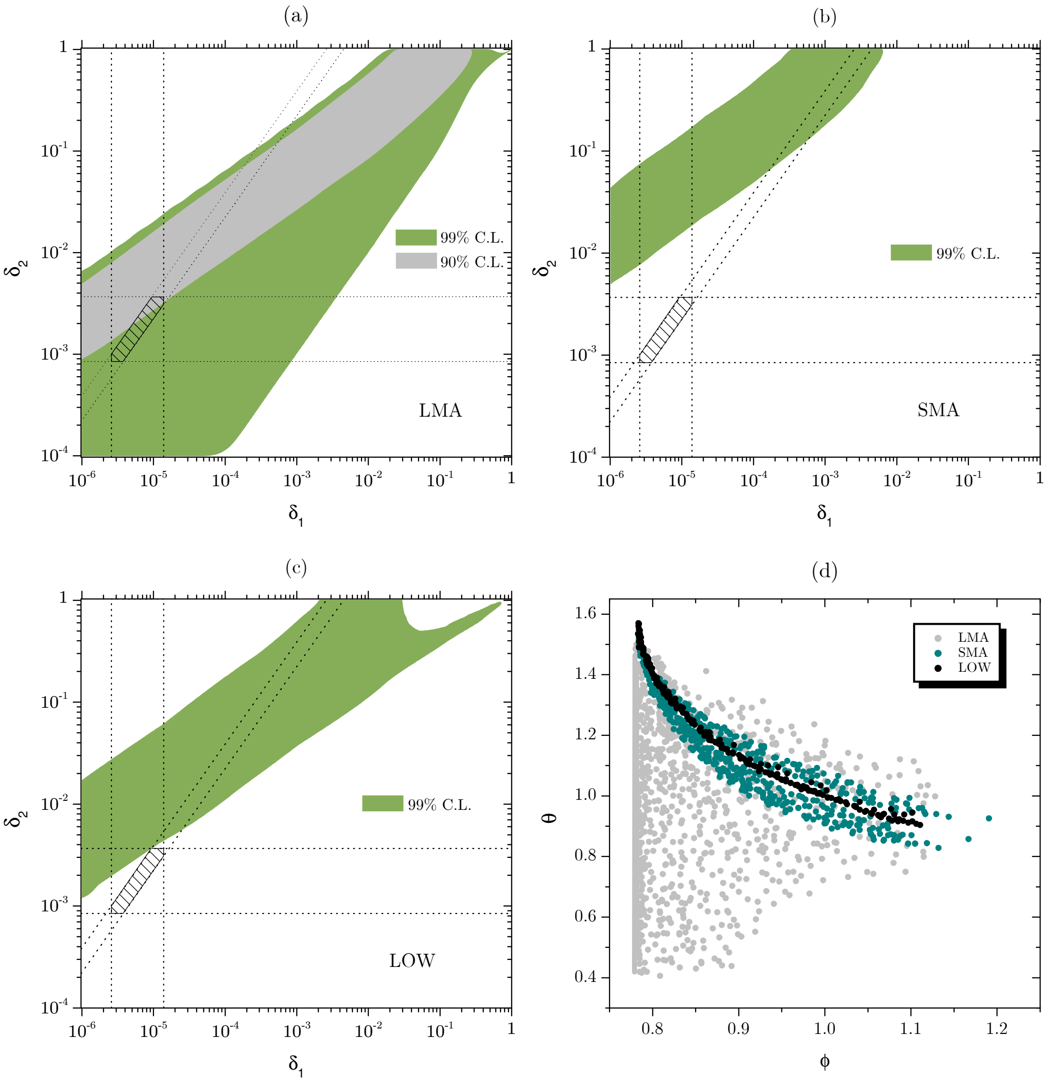

Figure 2: Allowed region in the plane (δ_1, δ_2) for the three different MSW solar solutions (LMA, SMA, LOW) at 99% C.L., if we assume $\mathbf{Y}_\nu \propto \mathbf{Y}_U$ and an exactly degenerate spectrum for the right-handed Majorana neutrinos at the unification scale Λ_X. The dashed area corresponds to the allowed region for the up quark mass ratios at Λ_X. At 90% C.L., the only solution that is compatible with the experimental data is the LMA one. The corresponding allowed region in the plane (ϕ, θ) is shown in figure 2d at 99% C.L..

where $\eta_i^{CP} = \pm 1$ are the neutrino CP parities. By examining the invariants of the effective neutrino mass matrix given in eq. (3.25), one can conclude that one of the neutrinos should have a negative CP parity and the other two should have positive CP parity. This is verified for the numerical examples considered in table 2, where the neutrino with mass m_2 has $\eta_2^{CP} = -1$, while the other two have $\eta_{1,3}^{CP} = +1$. In this case, eq. (4.10) reads

$$|\langle m \rangle| \ = \left| m_1 \, U_{e1}^2 - m_2 \, U_{e2}^2 + m_3 \, U_{e3}^2 \right| \ . \tag{4.11}$$

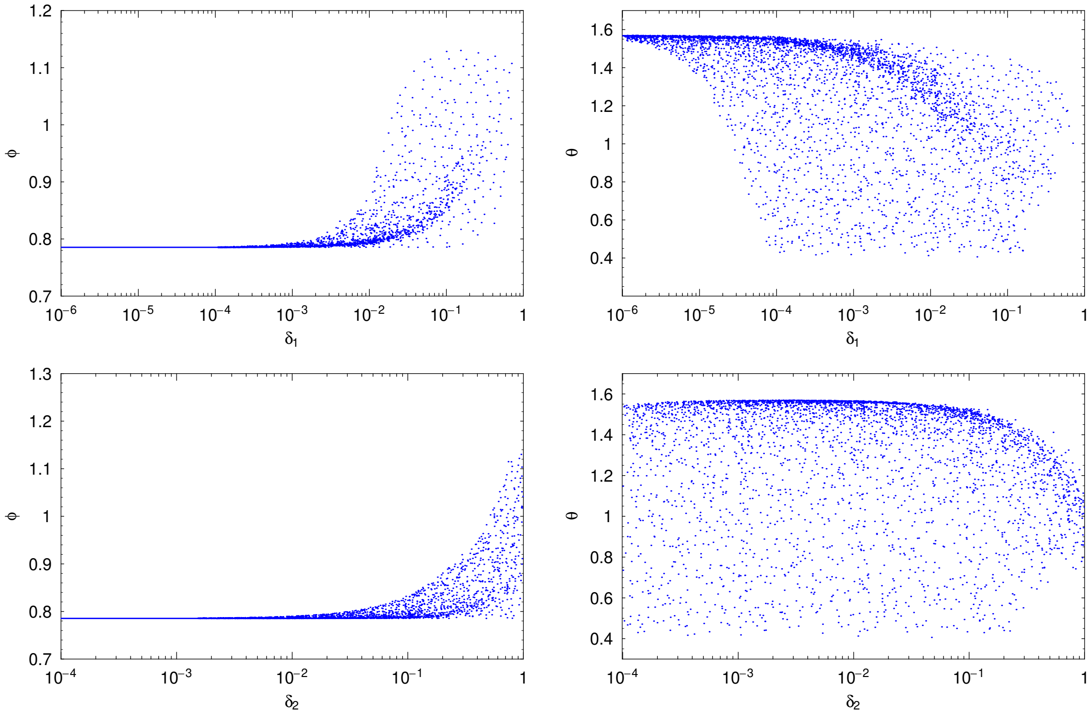

Figure 3: Allowed region at 99% C.L. for the most favoured solar solution, namely the LMA MSW solution, in different planes of the input parameters δ_1, δ_2, ϕ and θ, within our framework. For small values of the parameters δ_1 and δ_2, the angles ϕ and θ approach $\pi/4$ and $\pi/2$, respectively.

A particular feature in the above expression is the possibility of a cancellation among the different terms [31, 26], such that $|\langle m \rangle| \simeq 0$. In fact, for all the numerical cases considered in table 2, such a cancellation indeed occurs. This can be easily understood if we recall that the effective Majorana mass parameter is nothing but the absolute value of $(\mathbf{M}_\nu^{\rm eff})_{11}$. Therefore, from eq. (3.25) we have

$$|\langle m \rangle| \;\propto |\, \delta_1^2 \, (c_\theta^2 + c_{2\phi}\, s_\theta^2) \,| \, . \tag{4.12}$$

Since $\theta \simeq \pi/2$ and $\phi \simeq \pi/4$, in order to account for the low-energy neutrino data, and δ_1 is small, it is now clear from eq. (4.12) why $|\langle m \rangle|$ is highly suppressed in our framework.

5. A simple analytical form for $\mathbf{M}_\nu^{\rm eff}$

In this section we shall present a simple analytical example which can reproduce the results obtained in the last section. We have seen that, in the context of complete degeneracy of right-handed neutrinos and imposing proportionality between $\mathbf{Y}_\nu$ and $\mathbf{Y}_U$, the only possible solution at low energies is the LMA. Moreover, as it is clear from figures 3 this would require $\theta \simeq \pi/2$ and $\phi \simeq \pi/4$. On the other hand, if one

	$\mathbf{Y}_\nu = \mathbf{Y}_U$	$\mathbf{Y}_\nu = 3\,\mathbf{Y}_U$	**Best fit**
At $\Lambda_X = 2.0\times10^{16}$ GeV			
y_u	5.0×10^{-6}	5.9×10^{-6}	5.6×10^{-6}
y_c	1.7×10^{-3}	2.1×10^{-3}	1.7×10^{-3}
y_t	0.68	0.80	0.64
y_1	5.0×10^{-6}	1.8×10^{-5}	6.4×10^{-7}
y_2	1.7×10^{-3}	6.3×10^{-3}	1.7×10^{-3}
y_3	0.68	2.4	0.64
$\pi/2-\theta$	7.0×10^{-3}	7.0×10^{-3}	2.6×10^{-3}
$\phi-\pi/4$	1.0×10^{-5}	1.0×10^{-5}	2.3×10^{-6}
At Λ_R =	2.6×10^{9} GeV	1.4×10^{11} GeV	6.3×10^{8} GeV
M_1 (GeV)	2.4×10^{9}	9.6×10^{10}	5.8×10^{8}
M_2 (GeV)	$(-)\,2.4\times10^{9}$	$(-)\,9.6\times10^{10}$	$(-)\,5.8\times10^{8}$
M_3 (GeV)	2.6×10^{9}	1.4×10^{11}	6.3×10^{8}
r_ν	6.7×10^{-2}	8.1×10^{-2}	1.4×10^{-2}
$\tan^2\theta_{12}$	0.34	0.34	0.26
$\tan^2\theta_{23}$	0.47	0.57	1.4
$\lvert U_{e3}\rvert$	0.18	0.18	0.05
At $M_Z = 91.2\,$GeV			
$K_{\rm eff}$	0.31	2.18	0.30
m_1 (eV)	1.6×10^{-3}	1.9×10^{-3}	1.6×10^{-3}
m_2 (eV)	$(-)\,1.0\times10^{-2}$	$(-)\,1.1\times10^{-2}$	$(-)\,6.8\times10^{-3}$
m_3 (eV)	4.0×10^{-2}	3.9×10^{-2}	5.6×10^{-2}
Δm^2_{12} (eV^2)	1.0×10^{-4}	1.2×10^{-4}	4.3×10^{-5}
Δm^2_{23} (eV^2)	1.5×10^{-3}	1.4×10^{-3}	3.1×10^{-3}
r_ν	6.7×10^{-2}	8.1×10^{-2}	1.4×10^{-2}
$\tan^2\theta_{12}$	0.34	0.34	0.26
$\tan^2\theta_{23}$	0.47	0.57	1.4
$\lvert U_{e3}\rvert$	0.18	0.18	0.05
$\lvert\langle m\rangle\rvert$ (eV)	2.0×10^{-12}	2.4×10^{-12}	9.8×10^{-15}
m_u (MeV)	1.9	2.0	2.2
m_c (GeV)	0.68	0.69	0.66
m_t (GeV)	174	174	175
Down-quark masses	$m_d = 4.61\,$MeV	$m_s = 91.8\,$MeV	$m_b = 2.99\,$GeV
Charged-lepton masses	$m_e = 0.49\,$MeV	$m_\mu = 102.8\,$MeV	$m_\tau = 1.75\,$GeV

Table 2: Numerical results for $\tan\beta = 10$. We assume right-handed neutrino degeneracy at Λ_X, which is then lifted by radiative corrections. Two different Ansätze were considered for the Dirac neutrino Yukawa coupling matrix, $\mathbf{Y}_\nu = \mathbf{Y}_U$ and $\mathbf{Y}_\nu = 3\mathbf{Y}_U$. By slightly modifying the initial condition $y_1/y_3 = y_u/y_t$ it is possible to reproduce the best-fit values for all the solar and atmospheric neutrino data, as shown in the last column of the table. The negative CP parities are indicated by $(-)$.

assumes in eq. (3.25), $\phi = \pi/4$ and $\theta = \pi/2$, then

$$\boldsymbol{\kappa}(M_Z) = \frac{K_{\rm eff}}{\Lambda_R}\begin{pmatrix} 0 & 0 & -\delta_1 \\ 0 & \delta_2^2 & 0 \\ -\delta_1 & 0 & 0 \end{pmatrix}. \tag{5.1}$$

In this case the neutrino masses are

$$\{ m_1 \, , \, m_2 \, , \, m_3 \} = \frac{K_{\rm eff}}{\Lambda_R} \, v^2 \sin^2\beta \, \{ \delta_1 \, , \, \delta_1 \, , \, \delta_2^2 \} \, . \tag{5.2}$$

Taking into account eqs. (5.1) and (5.2), it is straightforward to show that, independently of the value of the parameters δ_1 and δ_2, one always has $\sin^2(2\,\theta_{23}) = 0$, which is not phenomenologically viable.

At first sight the above result may seem contradictory, since on one hand our numerical study clearly shows that values of $\theta \simeq \pi/2$ and $\phi \simeq \pi/4$ are indeed compatible with the low-energy neutrino data, but on the other hand, a simple analytical check gives a completely opposite answer. As we shall see shortly, the reason for this mismatching lies in the fact that small deviations, such as $\delta\theta = \pi/2 - \theta \ll 1$ and $\delta\phi = \phi - \pi/4 \ll 1$, and/or small perturbations from our original assumptions, turn out to be crucial. To show that this is indeed the case, let us relax the condition of proportionality between $\mathbf{Y}_\nu$ and $\mathbf{Y}_U$ for the smallest Dirac neutrino Yukawa coupling by putting

$$\epsilon_1 = a\,\epsilon^2 \, , \qquad \epsilon_2 = \epsilon = \frac{y_c(\Lambda_X)}{y_t(\Lambda_X)} \, , \tag{5.3}$$

and write θ and ϕ as

$$\phi = \frac{\pi}{4} + b\,\epsilon^2 \, , \qquad \theta = \frac{\pi}{2} - \epsilon \, , \tag{5.4}$$

where a and b are some positive real constants. From eq. (3.23) we have then

$$\delta_1 = a \, I_{\rm eff} \, I_\tau^{-1} \, \epsilon^2 \simeq a\,\epsilon^2 \, , \qquad \delta_2 = I_{\rm eff} \, I_\tau^{-1} \, \epsilon \simeq \epsilon \, . \tag{5.5}$$

The effective neutrino mass operator (3.25) will be approximately given in this case by

$$\boldsymbol{\kappa}(M_Z) \simeq \frac{K_{\rm eff}}{\Lambda_R} \, \epsilon^2 \begin{pmatrix} a^2\,\epsilon^4\,(\,1 - 2\,b\,) & -a\,\epsilon^2 & -a \\ -a\,\epsilon^2 & 1 - 2\,b\,\epsilon^4 & -1 \\ -a & -1 & 2\,b \end{pmatrix} . \tag{5.6}$$

Due to the smallness of the parameter ϵ we have, to a very good approximation,

$$\boldsymbol{\kappa}(M_Z) \simeq \frac{K_{\rm eff}}{\Lambda_R} \, \epsilon^2 \, \boldsymbol{\kappa}' \, , \qquad \boldsymbol{\kappa}' = \begin{pmatrix} 0 & 0 & -a \\ 0 & 1 & -1 \\ -a & -1 & 2\,b \end{pmatrix} . \tag{5.7}$$

The invariants of the matrix $\boldsymbol{\kappa}'$ are

$$\begin{aligned} \mathrm{T} &= \lambda_1 + \lambda_2 + \lambda_3 = 1 + 2\,b \, , \\ \chi &= \lambda_1\,\lambda_2 + \lambda_1\,\lambda_3 + \lambda_2\,\lambda_3 = -1 - a^2 - 2\,b \, , \\ \Delta &= \lambda_1\,\lambda_2\,\lambda_3 = -a^2 \, , \end{aligned} \tag{5.8}$$

where λ_i are the eigenvalues,

$$\lambda_1 = 2\sqrt{p}\,\cos\left(\frac{\alpha}{3}\right) + \mathrm{T}\,,$$
$$\lambda_{2,3} = -2\sqrt{p}\,\cos\left(\frac{\alpha}{3}\pm\frac{\pi}{3}\right) + \mathrm{T}\,, \tag{5.9}$$

with

$$q = -\frac{2\,\mathrm{T}^3}{27} + \frac{\mathrm{T}\,\chi}{3} - \Delta\,, \qquad p = \frac{\mathrm{T}^2}{9} - \frac{\chi}{3}\,, \qquad \cos\alpha = -\frac{q}{p^{3/2}}\,. \tag{5.10}$$

The corresponding eigenvectors are given by

$$\mathbf{v}_i = \frac{1}{n_i}\left[a\,(1-\lambda_i)\,,\ -\lambda_i\,,\ \lambda_i\,(\lambda_i - 1)\right]^T\,, \tag{5.11}$$

where n_i are the normalization coefficients.

In figure 4 we present the region in the (a, b)-plane, which is compatible with the low-energy solar and atmospheric neutrino data at 90% and 99% C.L. for the LMA solution (cf. table 1). We see that no fine-tuning is required in order to reproduce the data. To obtain the best-fit point (black dot in the figure), we assume at the unification scale Λ_X the same values for the charm and top Yukawa couplings as in the last column of table 2, i.e. $y_c(\Lambda_X) = 1.7\times10^{-3}$ and $y_t(\Lambda_X) = 0.64$. Taking then as input parameters

$$\epsilon = \frac{y_c(\Lambda_X)}{y_t(\Lambda_X)} = 2.65\times10^{-3}\,, \qquad a = 0.15\,, \qquad b = 0.34\,, \qquad \Lambda_R = 6.3\times10^{8}\,\mathrm{GeV}\,, \tag{5.12}$$

the following values are obtained at the electroweak scale for $\tan\beta = 10$:

$$\begin{aligned}
K_{\rm eff} &= 0.3\,, & \delta_1 &= 1.0\times10^{-6}\,, & \delta_2 &= 2.7\times10^{-3}\,,\\
m_1 &= 1.6\times10^{-3}\,\mathrm{eV}\,, & m_2 &= 6.7\times10^{-3}\,\mathrm{eV}\,, & m_3 &= 5.6\times10^{-2}\,\mathrm{eV}\,,\\
\Delta m^2_{12} &= 4.2\times10^{-5}\ \mathrm{eV}^2\,, & \Delta m^2_{23} &= 3.1\times10^{-3}\ \mathrm{eV}^2\,, & r_\nu &= 1.4\times10^{-2}\,,\\
\tan^2\theta_{12} &= 0.26\,, & \tan^2\theta_{23} &= 1.4\,, & |U_{e3}| &= 0.05\,,\\
|\langle m\rangle| &= 9.3\times10^{-15}\,\mathrm{eV}\,,
\end{aligned} \tag{5.13}$$

which correspond to the best-fit values in the neutrino mixing plane for the atmospheric neutrinos [27] and the LMA solar neutrino solution [28].

6. Conclusions

In this paper we have considered the possibility of having an exactly degenerate spectrum for heavy right-handed Majorana neutrinos at the grand unification scale. Our theoretical framework was based on the minimal supersymmetric standard model

JHEP09(2001)015

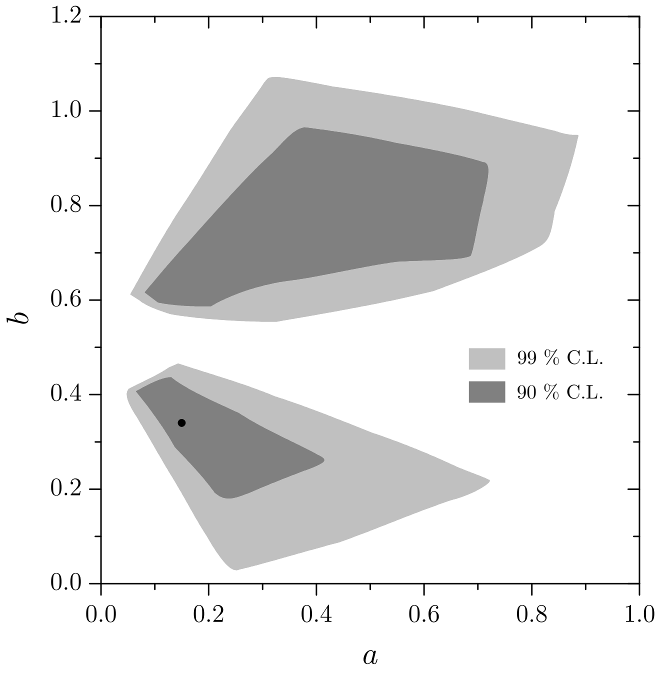

Figure 4: Allowed region for the parameters a and b of the triangular texture (5.7) for $\boldsymbol{\kappa}(M_Z)$, in the case of the LMA MSW solution and considering the neutrino data at 90% and 99% C.L. summarized in table 1. The black dot corresponds to the point which leads to the best-fit values of the solar and atmospheric data as shown in eqs. (5.12)–(5.13).

with unbroken R parity and extended with three heavy Majorana neutrino fields. We assumed no Dirac-type leptonic mixing at GUT scale, and thus the right-handed neutrino sector constitutes the only source of lepton flavour violation in this context. We have then studied the renormalization group effects, which include the neutrino threshold effects, from high to low energies.

Taking into account the allowed range for the quark mass ratios at GUT scale and the effective neutrino mass matrix obtained through the seesaw mechanism, we have investigated which of the possible solutions to the solar neutrino problem can be accommodated in our framework. Inspired by GUT-motivated relations among the quark, charged-lepton and Dirac neutrino mass matrices, and based on the latest global analyses of the solar and atmospheric data at 99% C.L., we have then concluded that the only solar solutions compatible with the experimental data are the LOW and LMA solutions. In fact, at 90% C.L. only the latter is allowed. Even in this case, the obtained values for the neutrino oscillation parameters are far from the best-fit points. In this sense, right-handed neutrino degeneracy embedded in GUT scenarios seems to be disfavoured by the low-energy neutrino data. We emphasize however that, by slightly relaxing the imposed relations among the Yukawa coupling matrices at high scale, one can easily reach the best-fit values. Our assumption of degeneracy is in fact a very restrictive one, since the right-handed Majorana neutrino

JHEP09(2001)015

matrix $\mathbf{M}_R$ is parametrized in terms of only two angles and an overall scale Λ_R. On the other hand, a fully or partially hierarchical mass spectrum for right-handed neutrinos can be easily achieved with a much less constrained structure, and one could therefore expect that such patterns are easier to be reconciled with the low-energy neutrino data.

Finally, we are also aware of possible contributions to lepton flavour violation processes like $\mu \to e\gamma$. As it has been recently pointed out by Casas and Ibarra [32], in supersymmetric theories with seesaw-like neutrino masses the prediction for the branching ratio of this process is, in general, larger than the experimental upper bound. In particular, scenarios such as the one considered here, where top-neutrino unification occurs, are severely constrained unless the leptonic Yukawa coupling matrices satisfy very specific requirements.

Acknowledgments

We thank E.Kh. Akhmedov for a careful reading of the manuscript and useful comments. We are also grateful to D.F. Carvalho for bringing to our attention some aspects of lepton flavour violation in $\mu \to e\gamma$. The work of R.G.F. and F.R.J. has been supported by *Fundação para a Ciência e a Tecnologia* under the grants SFRH/BPD/1549/2000 and PRAXISXXI/BD/18219/98, respectively.

A. Renormalization group equations

In this appendix we present the relevant one-loop renormalization group equations for the MSSM extended with three right-handed Majorana neutrinos.

Between Λ_X and Λ_R the evolution of the up-quark, $\mathbf{Y}_U$, down-quark, $\mathbf{Y}_D$, charged-lepton, $\mathbf{Y}_\ell$, and Dirac neutrino, $\mathbf{Y}_\nu$, Yukawa coupling matrices is given by

$$16\,\pi^2 \frac{d\mathbf{Y}_U}{dt} = \Big[(T_U - G_U) + (3\,\mathbf{Y}_U\mathbf{Y}_U^\dagger + \mathbf{Y}_D\mathbf{Y}_D^\dagger)\Big]\,\mathbf{Y}_U\,, \tag{A.1}$$

$$16\,\pi^2 \frac{d\mathbf{Y}_D}{dt} = \Big[(T_D - G_D) + (3\,\mathbf{Y}_D\mathbf{Y}_D^\dagger + \mathbf{Y}_U\mathbf{Y}_U^\dagger)\Big]\,\mathbf{Y}_D\,, \tag{A.2}$$

$$16\,\pi^2 \frac{d\mathbf{Y}_\ell}{dt} = \Big[(T_\ell - G_\ell) + (3\,\mathbf{Y}_\ell\mathbf{Y}_\ell^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\Big]\,\mathbf{Y}_\ell\,, \tag{A.3}$$

$$16\,\pi^2 \frac{d\mathbf{Y}_\nu}{dt} = \Big[(T_\nu - G_\nu) + (3\,\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\ell\mathbf{Y}_\ell^\dagger)\Big]\,\mathbf{Y}_\nu\,, \tag{A.4}$$

where $t = \ln(\mu/M_Z)$, μ is the renormalization scale. The quantities T_i and G_i are defined as

$$T_U = \mathrm{Tr}(3\,\mathbf{Y}_U\mathbf{Y}_U^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\,, \qquad G_U = \frac{16}{3}\,g_3^2 + 3\,g_2^2 + \frac{13}{15}\,g_1^2\,, \tag{A.5}$$

$$T_D = \mathrm{Tr}(3\,\mathbf{Y}_D\mathbf{Y}_D^\dagger + \mathbf{Y}_\ell\mathbf{Y}_\ell^\dagger)\,, \qquad G_D = \frac{16}{3}\,g_3^2 + 3\,g_2^2 + \frac{7}{15}\,g_1^2\,, \tag{A.6}$$

$$T_\ell = \mathrm{Tr}(3\,\mathbf{Y}_D\mathbf{Y}_D^\dagger + \mathbf{Y}_\ell\mathbf{Y}_\ell^\dagger)\,, \qquad G_\ell = 3\,g_2^2 + \frac{9}{15}\,g_1^2\,, \tag{A.7}$$

$$T_\nu = \mathrm{Tr}(3\,\mathbf{Y}_U\mathbf{Y}_U^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\,, \qquad G_\nu = 3\,g_2^2 + \frac{3}{5}\,g_1^2\,, \tag{A.8}$$

where g_3, g_2, and g_1 are the SU(3), SU(2) and U(1) gauge couplings, which satisfy the RGE

$$16\,\pi^2\frac{d\,g_i}{dt} = b_i\,g_i^3\,, \qquad b_i = (33/5, 1, -3)\,. \tag{A.9}$$

The evolution of the right-handed neutrino mass matrix, $\mathbf{M}_R$, is given by

$$8\,\pi^2\frac{d\mathbf{M}_R}{dt} = \mathbf{M}_R\,(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)^T + (\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\,\mathbf{M}_R\,. \tag{A.10}$$

Below Λ_R, the renormalization group equations for the up-quark and charged-lepton Yukawa couplings get modified since after decoupling of the right-handed neutrino states, the contribution of $\mathbf{Y}_\nu$ should vanish. In this case the RGE for $\mathbf{Y}_U$ and $\mathbf{Y}_\ell$ read

$$16\,\pi^2\frac{d\mathbf{Y}_U}{dt} = \left[(T'_U - G_U) + (3\,\mathbf{Y}_U\mathbf{Y}_U^\dagger + \mathbf{Y}_D\mathbf{Y}_D^\dagger)\right]\mathbf{Y}_U\,, \tag{A.11}$$

$$16\,\pi^2\frac{d\mathbf{Y}_\ell}{dt} = \left[(T_\ell - G_\ell) + 3\,\mathbf{Y}_\ell\mathbf{Y}_\ell^\dagger\right]\mathbf{Y}_\ell\,, \tag{A.12}$$

with

$$T'_U = 3\,\mathrm{Tr}(\mathbf{Y}_U\mathbf{Y}_U^\dagger)\,. \tag{A.13}$$

Finally, the evolution of the effective neutrino mass matrix $\boldsymbol{\kappa}$ from Λ_R to M_Z is given by

$$16\,\pi^2\frac{d\boldsymbol{\kappa}}{dt} = (T_\kappa - G_\kappa)\boldsymbol{\kappa} + (\mathbf{Y}_\ell\mathbf{Y}_\ell^\dagger)\,\boldsymbol{\kappa} + \boldsymbol{\kappa}\,(\mathbf{Y}_\ell\mathbf{Y}_\ell^\dagger)^T\,, \tag{A.14}$$

where

$$T_\kappa = 6\,\mathrm{Tr}(\mathbf{Y}_U\mathbf{Y}_U^\dagger)\,, \qquad G_\kappa = 6\,g_2^2 + \frac{6}{5}\,g_1^2\,. \tag{A.15}$$

JHEP09(2001)015

References

[1] Super-Kamiokande collaboration, *Measurement of a small atmospheric ν_μ/ν_e ratio*, *Phys. Lett.* **B 433** (1998) 9 [hep-ex/9803006]; *Study of the atmospheric neutrino flux in the multi-GeV energy range*, *Phys. Lett.* **B 436** (1998) 33 [hep-ex/9805006]; SuperKamiokande collaboration, *Neutrino-induced upward stopping muons in super-Kamiokande*, *Phys. Lett.* **B 467** (1999) 185 [hep-ex/9908049]; *Measurement of the flux and zenith-angle distribution of upward through-going muons by super-Kamiokande*, *Phys. Rev. Lett.* **82** (1999) 2644 [hep-ex/9812014];
Tau neutrinos favored over sterile neutrinos in atmospheric muon neutrino oscillations, *Phys. Rev. Lett.* **85** (2000) 3999 [hep-ex/0009001].

[2] CHOOZ collaboration, *Initial results from the CHOOZ long baseline reactor neutrino oscillation experiment*, *Phys. Lett.* **B 420** (1998) 397 [hep-ex/9711002]; *Limits on neutrino oscillations from the CHOOZ experiment*, *Phys. Lett.* **B 466** (1999) 415 [hep-ex/9907037].

[3] Super-Kamiokande collaboration, *Measurements of the solar neutrino flux from super-Kamiokande's first 300 days*, *Phys. Rev. Lett.* **81** (1998) 1158 [hep-ex/9805021], erratum *ibid.* **81** (1998) 4279; *Constraints on neutrino oscillation parameters from the measurement of day-night solar neutrino fluxes at super-Kamiokande*, *Phys. Rev. Lett.* **82** (1999) 1810 [hep-ex/9812009];
Measurement of the solar neutrino energy spectrum using neutrino electron scattering, *Phys. Rev. Lett.* **82** (1999) 2430 [hep-ex/9812011];
Solar 8B and hep neutrino measurements from 1258 days of super-Kamiokande data, *Phys. Rev. Lett.* **86** (2001) 5651 [hep-ex/0103032];
Constraints on neutrino oscillations using 1258 days of super-Kamiokande solar neutrino data, *Phys. Rev. Lett.* **86** (2001) 5656 [hep-ex/0103033].

[4] K. Lande, T. Daily, R. Davis, J.R. Distel, B.T. Clevland, C.K. Lee, P.S. Wildenhain and J. Ullman, *Measurement of the solar electron neutrino flux with the Homestake Chlorine detector*, *Astrophys. J.* **496** (1998) 505;
R. Davis, *A review of the Homestake solar neutrino experiment*, *Prog. Part. Nucl. Phys.* **32** (1994) 13.

[5] SAGE collaboration, *Measurement of the solar neutrino capture rate with gallium metal*, *Phys. Rev.* **C 60** (1999) 055801 [astro-ph/9907113].

[6] GALLEX collaboration, *Gallex solar neutrino observations: results for Gallex IV*, *Phys. Lett.* **B 447** (1999) 127.

[7] L. Wolfenstein, *Neutrino oscillations in matter*, *Phys. Rev.* **D 17** (1978) 2369;
S.P. Mikheev and A.Y. Smirnov, *Resonance enhancement of oscillations in matter and solar neutrino spectroscopy*, *Sov. J. Nucl. Phys.* **42** (1985) 913.

[8] M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories* in *Supergravity*, P. van Nieuwenhuizen and D.Z. Freedman eds., North Holland, Ams-

terdam, 1979, p. 315;
T. Yanagida, *Horizontal symmetry and masses of neutrinos* in *Proc. of the Workshop on the unified theory and baryon number in the universe*, KEK report 79-18 (1979), p. 95;
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity violation*, *Phys. Rev. Lett.* **44** (1980) 912.

[9] For a recent review on grand unified theories see e.g. R.N. Mohapatra, *Supersymmetric grand unification: an update*, Particle Physics, Trieste, Italy, 7 June - 9 July 1999, `hep-ph/9911272` in and references therein.

[10] H. Georgi and C. Jarlskog, *A new lepton-quark mass relation in a unified theory*, *Phys. Lett.* **B 86** (1979) 297.

[11] J.A. Harvey, P. Ramond and D.B. Reiss, *CP-violation and mass relations in* SO(10), *Phys. Lett.* **B 92** (1980) 309;
X.G. He and S. Meljanac, *Symmetry breaking and mass spectra in supersymmetric* SO(10) *models*, *Phys. Rev.* **D 41** (1990) 1620;
S. Dimopoulos, L.J. Hall and S. Raby, *A predictive framework for fermion masses in supersymmetric theories*, *Phys. Rev. Lett.* **68** (1992) 1984;
K.S. Babu and R.N. Mohapatra, *Mass matrix textures from superstring inspired* SO(10) *models*, *Phys. Rev. Lett.* **74** (1995) 2418 [`hep-ph/9410326`].

[12] A.Y. Smirnov, *Seesaw enhancement of lepton mixing*, *Phys. Rev.* **D 48** (1993) 3264 [`hep-ph/9304205`].

[13] G.K. Leontaris, S. Lola, C. Scheich and J.D. Vergados, *Textures for neutrino mass matrices*, *Phys. Rev.* **D 53** (1996) 6381 [`hep-ph/9509351`];
J.R. Ellis, G.K. Leontaris, S. Lola and D.V. Nanopoulos, *Neutrino textures in the light of super-Kamiokande data and a realistic string model*, *Eur. Phys. J.* **C 9** (1999) 389 [`hep-ph/9808251`].

[14] P. Binetruy, S. Lavignac, S.T. Petcov and P. Ramond, *Quasi-degenerate neutrinos from an abelian family symmetry*, *Nucl. Phys.* **B 496** (1997) 3 [`hep-ph/9610481`].

[15] E.K. Akhmedov, G.C. Branco and M.N. Rebelo, *Seesaw mechanism and structure of neutrino mass matrix*, *Phys. Lett.* **B 478** (2000) 215 [`hep-ph/9911364`].

[16] E.K. Akhmedov, G.C. Branco, F.R. Joaquim and J.I. Silva-Marcos, *Neutrino masses and mixing with seesaw mechanism and universal breaking of extended democracy*, *Phys. Lett.* **B 498** (2001) 237 [`hep-ph/0008010`].

[17] G. Altarelli and F. Feruglio, *A simple grand unification view of neutrino mixing and fermion mass matrices*, *Phys. Lett.* **B 451** (1999) 388 [`hep-ph/9812475`];
C.H. Albright and S.M. Barr, *Realization of the large mixing angle solar neutrino solution in an* SO(10) *supersymmetric grand unified model*, *Phys. Rev.* **D 64** (2001) 073010 [`hep-ph/0104294`].

JHEP09(2001)015

[18] J.R. Ellis and S. Lola, *Can neutrinos be degenerate in mass?*, *Phys. Lett.* **B 458** (1999) 310 [hep-ph/9904279];
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *Nearly degenerate neutrinos, supersymmetry and radiative corrections*, *Nucl. Phys.* **B 569** (2000) 82 [hep-ph/9905381].

[19] A.S. Dighe and A.S. Joshipura, *No-go for exactly degenerate neutrinos at high scale?*, hep-ph/0010079.

[20] E.J. Chun, *Lepton flavor violation and radiative neutrino masses*, *Phys. Lett.* **B 505** (2001) 155 [hep-ph/0101170].

[21] P.H. Chankowski and Z. Pluciennik, *Renormalization group equations for seesaw neutrino masses*, *Phys. Lett.* **B 316** (1993) 312 [hep-ph/9306333];
K.S. Babu, C.N. Leung and J. Pantaleone, *Renormalization of the neutrino mass operator*, *Phys. Lett.* **B 319** (1993) 191 [hep-ph/9309223];
M. Tanimoto, *Renormalization effect on large neutrino flavor mixing in the minimal supersymmetric standard model*, *Phys. Lett.* **B 360** (1995) 41 [hep-ph/9508247];
J.R. Ellis and S. Lola, *Can neutrinos be degenerate in mass?*, *Phys. Lett.* **B 458** (1999) 310 [hep-ph/9904279];
N. Haba, N. Okamura and M. Sugiura, *The renormalization group analysis of the large lepton flavor mixing and the neutrino mass*, *Prog. Theor. Phys.* **103** (2000) 367 [hep-ph/9810471];
N. Haba, Y. Matsui, N. Okamura and M. Sugiura, *Energy scale dependence of the lepton flavor mixing matrix*, *Eur. Phys. J.* **C 10** (1999) 677 [hep-ph/9904292];
N. Haba and N. Okamura, *Stability of the lepton-flavor mixing matrix against quantum corrections*, *Eur. Phys. J.* **C 14** (2000) 347 [hep-ph/9906481];
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *Naturalness of nearly degenerate neutrinos*, *Nucl. Phys.* **B 556** (1999) 3 [hep-ph/9904395];
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *Theoretical constraints on the vacuum oscillation solution to the solar neutrino problem*, *J. High Energy Phys.* **09** (1999) 015 [hep-ph/9906281];
General RG equations for physical neutrino parameters and their phenomenological implications, *Nucl. Phys.* **B 573** (2000) 652 [hep-ph/9910420];
E. Ma, *Stability of neutrino mass degeneracy*, *J. Phys.* **G 25** (1999) L97 [hep-ph/9907400];
R. Barbieri, G.G. Ross and A. Strumia, *Vacuum oscillations of quasi degenerate solar neutrinos*, *J. High Energy Phys.* **10** (1999) 020 [hep-ph/9906470];
P.H. Chankowski, W. Krolikowski and S. Pokorski, *Fixed points in the evolution of neutrino mixings*, *Phys. Lett.* **B 473** (2000) 109 [hep-ph/9910231];
S.F. King and N.N. Singh, *Renormalisation group analysis of single right-handed neutrino dominance*, *Nucl. Phys.* **B 591** (2000) 3 [hep-ph/0006229].

[22] G.C. Branco, M.N. Rebelo and J.I. Silva-Marcos, *Degenerate and quasidegenerate Majorana neutrinos*, *Phys. Rev. Lett.* **82** (1999) 683 [hep-ph/9810328].

[23] S. Weinberg, *Baryon and lepton nonconserving processes*, *Phys. Rev. Lett.* **43** (1979) 1566.

[24] K.S. Babu, C.N. Leung and J. Pantaleone, *Renormalization of the neutrino mass operator*, *Phys. Lett.* **B 319** (1993) 191 [hep-ph/9309223].

[25] Particle Data Group collaboration, *Review of particle physics*, *Eur. Phys. J.* **C 15** (2000) 1.

[26] S.M. Bilenky, S. Pascoli and S.T. Petcov, *Majorana neutrinos, neutrino mass spectrum, CP-violation and neutrinoless double beta-decay, 1. The three-neutrino mixing case*, *Phys. Rev.* **D 64** (2001) 053010 [hep-ph/0102265].

[27] M.C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay and J.W.F. Valle, *Global three-neutrino oscillation analysis of neutrino data*, *Phys. Rev.* **D 63** (2001) 033005 [hep-ph/0009350].

[28] J.N. Bahcall, P.I. Krastev and A.Yu. Smirnov, *Solar neutrinos: global analysis and implications for SNO*, *J. High Energy Phys.* **05** (2001) 015 [hep-ph/0103179].

[29] H. Fusaoka and Y. Koide, *Updated estimate of running quark masses*, *Phys. Rev.* **D 57** (1998) 3986 [hep-ph/9712201].

[30] L. Baudis et al., *Limits on the majorana neutrino mass in the 0.1 eV range*, *Phys. Rev. Lett.* **83** (1999) 41 [hep-ex/9902014].

[31] L. Wolfenstein, *CP properties of Majorana neutrinos and double beta decay*, *Phys. Lett.* **B 107** (1981) 77.

[32] J.A. Casas and A. Ibarra, *Oscillating neutrinos and* $\mu \to e, \gamma$, hep-ph/0103065.